

Testing for a δ -neighborhood of a generalized Pareto copula process

Stefan Aulbach*

Department of Mathematics
University of Würzburg, Germany

Michael Falk

Department of Mathematics
University of Würzburg, Germany

Abstract: It is a well-known fact that a multivariate distribution function F is in the max-domain of attraction of an extreme value distribution if and only if this is true for the copula C corresponding to F and its univariate margins. Furthermore Aulbach et al. (2012) [1] have shown that a copula satisfies the extreme value condition if and only if the copula is tail equivalent to a generalized Pareto copula (GPC). A more specific condition, namely whether C is in some δ -neighborhood of a GPC, can be tested in arbitrary dimensions by means of a χ^2 -goodness-of-fit test. We propose an extension of this test to function space, considering copula processes \mathbf{U} in $C[0, 1]$, the set of all continuous functions defined on the compact unit interval. Therefore we test whether \mathbf{U} satisfies

$$P(\mathbf{U} \leq 1 + cf) = P(\mathbf{V} \leq 1 + cf) + O(c^{1+\delta}), \quad f \in E^-[0, 1],$$

as $c \downarrow 0$ where \mathbf{V} is a generalized Pareto copula process, $\delta > 0$, and $E^-[0, 1]$ is the set of all real valued and bounded functions on $[0, 1]$ attaining only non-positive values and having at most a countable set of discontinuities.

Key words and phrases: Functional max-domain of attraction, max-stable processes, D -norm, copula processes, sojourn time, chi-square goodness-of-fit test, multivariate max-domain of attraction, multivariate extreme value distribution, empirical copula, generalized Pareto copula.

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