Extremal properties of M4 processes *

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Abstract: Multivariate Maxima of Moving Maxima processes, or M4 for short, defined by

\[ X_{nj} = \bigvee_{l \geq 1} \bigvee_{-\infty < k < +\infty} \alpha_{l,k,j} Z_{l,n-k,j}, \ j = 1, \ldots, d, \ n \geq 1, \]

for nonnegative constants \( \{\alpha_{l,k,j}, l \geq 1, -\infty < k < +\infty, 1 \leq j \leq d\} \) satisfying \( \sum_{l=1}^{+\infty} \sum_{k=-\infty}^{+\infty} \alpha_{l,k,j} = 1, j = 1, \ldots, d, \) and \( \{Z_{l,n} = (Z_{l,n,1}, \ldots, Z_{l,n,d})\}_{l \geq 1}, -\infty < n < +\infty \) a sequence of independent random vectors with unit Fréchet margins, are very flexible models for temporally dependent multivariate extreme value processes.

The case where \( Z_{l,n,j} = Z_{l,n}, j = 1, \ldots, d, \) was considered by Smith and Weissman (1996). In contrast to this case, Martins and Ferreira (2005) considered the sequences \( \{Z_{l,n,j}\}_{l \geq 1}, -\infty < n < +\infty, j = 1, \ldots, d, \) to be independent, i.e., the vectors \( Z_{l,n} \) with independent margins.

We extend this M4 class by considering innovations \( Z_{l,n} \) with totally dependent margins for certain values of \( l - l \in I_1 \) and independent margins for the remaining values of \( l - l \in I_2 \). This produces a \( d \)-dimensional process whose extremal dependence, measured by the tail dependence coefficients, lies between the two previous cases. We study the extremal behaviour of such a process. Its extremal dependence is evaluated through the computation of its tail dependence coefficients and of the limiting distribution \( H \) of the vector \( \{M_{n1} = \bigvee_{i=1}^{n} X_{id}, \ldots, M_{nd} = \bigvee_{i=1}^{n} X_{id}\} \).

We compute the extremal index of \( \{X_n = (X_{n1}, \ldots, X_{nd})\}_{n \geq 1} \), which gives us information of the tendency for clustering of high values of this sequence as well as of its marginal sequences. The effect of the dimensions of \( I_1 \) and \( I_2 \) on the concordance of \( H \) is analyzed. We define and evaluate measures of the contagion effect of the occurrence of rare events in a margin of \( X_n, n \geq 1, \) on the same occurrences in the other margins.

Finally we illustrate the results by simulating the M4 processes considered and calculating the values of the different dependence coefficients.

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**References**
