Extremal properties of M4 processes *

H. Ferreira
Department of Mathematics
University of Beira Interior, Covilhã, Portugal

A.P. Martins
Department of Mathematics
University of Beira Interior, Covilhã, Portugal

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Abstract: Multivariate Maxima of Moving Maxima processes, or M4 for short, defined by

$$X_{nj} = \bigvee_{l\geq 1} \bigvee_{-\infty<k<+\infty} \alpha_{l,k,j} Z_{l,n-k,j},\ j = 1,\ldots,d,\ n \geq 1,$$

for nonnegative constants $\{\alpha_{l,k,j},\ l \geq 1,\ -\infty < k < +\infty,\ 1 \leq j \leq d\}$ satisfying $\sum_{l=1}^{+\infty}\sum_{k=-\infty}^{+\infty} \alpha_{l,k,j} = 1$, $j = 1,\ldots,d$, and $\{\mathbf{Z}_{l,n} = (Z_{l,n,1},\ldots,Z_{l,n,d})\}_{l\geq 1,\ -\infty<n<+\infty}$ a sequence of independent random vectors with unit Fréchet margins, are very flexible models for temporally dependent multivariate extreme value processes.

The case where $Z_{l,n,j} = Z_{l,n}$, $j = 1,\ldots,d$, was considered by Smith and Weissman (1996). In contrast to this case, Martins and Ferreira (2005) considered the sequences $\{Z_{l,n,j}\}_{l\geq 1,\ -\infty<n<+\infty}$, $j = 1,\ldots,d$, to be independent, *i.e.,* the vectors $\mathbf{Z}_{l,n}$ with independent margins.

We extend this M4 class by considering innovations $\mathbf{Z}_{l,n}$ with totally dependent margins for certain values of l - $l \in I_1$ - and independent margins for the remaining values of l - $l \in I_2$. This produces a $d-$dimensional process whose extremal dependence, measured by the tail dependence coefficients, lies between the two previous cases. We study the extremal behaviour of such a process. Its extremal dependence is evaluated through the computation of its tail dependence coefficients and of the limiting distribution H of the vector $(M_{n1} = \bigvee_{i=1}^{n} X_{id},\ldots,M_{nd} = \bigvee_{i=1}^{n} X_{id})$.

We compute the extremal index of $\{\mathbf{X}_n = (X_{n1},\ldots,X_{nd})\}_{n\geq 1}$, which gives us information of the tendency for clustering of high values of this sequence as well as of its marginal sequences. The effect of the dimensions of I_1 and I_2 on the concordance of H is analyzed. We define and evaluate measures of the contagion effect of the occurrence of rare events in a margin of $\mathbf{X}_n$, $n \geq 1$, on the same occurrences in the other margins.

Finally we illustrate the results by simulating the M4 processes considered and calculating the values of the different dependence coefficients.

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